# SCALING OF INDUCTANCE TO THE pH-LEVEL


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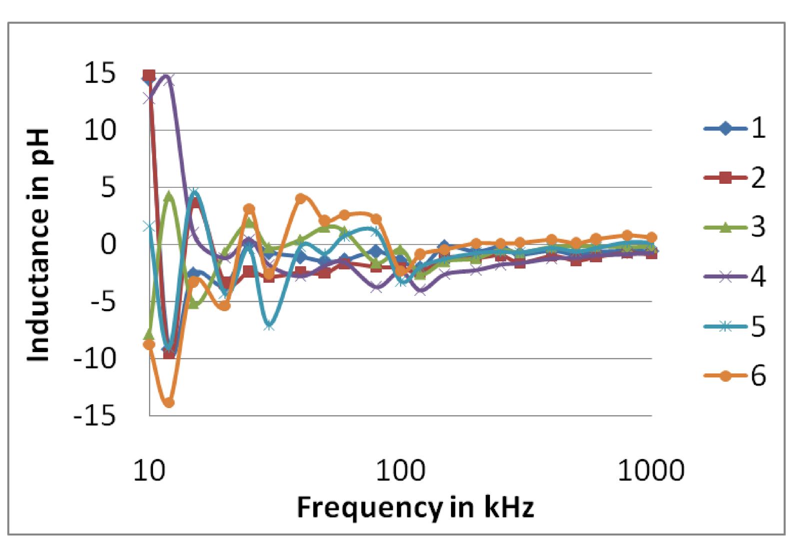
## Introduction

The demand for traceability of wideband power measurement is addressed in the iMERA+ project "The Next Generation of Power and Energy Measurements." One of the tasks is to improve the design of current shunts and comparison of the phase angle errors of different designs is needed. Within the project a wideband phase comparator for high current shunts has been developed by INRIM, see poster. SP has earlier shown that the absolute phase angle errors of 4-terminal current shunts can be determined by measuring the equivalent inductance of the shunts but for high current shunts the inductance can be <100 pH. Inductors with calculable value can easily be made on printed circuit board (PCB) in the range 10 nH to 100 nH but values <1 nH demand sub-mm design. This poster is a progress report on a scaling method to calibrate an impedance meter at the pH-level using inductance standards in the nH-range and a resistive voltage divider.

## Measurement setup

The measurements are made with a four-terminal pair impedance meter having a resolution <1 pH. To be able to connect 4-terminal current shunts and inductance standards an adapter from four-terminal pair (4TP) to four-terminal (4T) is used. The adapter is a 4TP test lead modified to have one voltage connector and one current connector that allow repeatability in the order of a few pH. A 4T short-circuit and an open-circuit are used for the impedance meters internal adjustment.



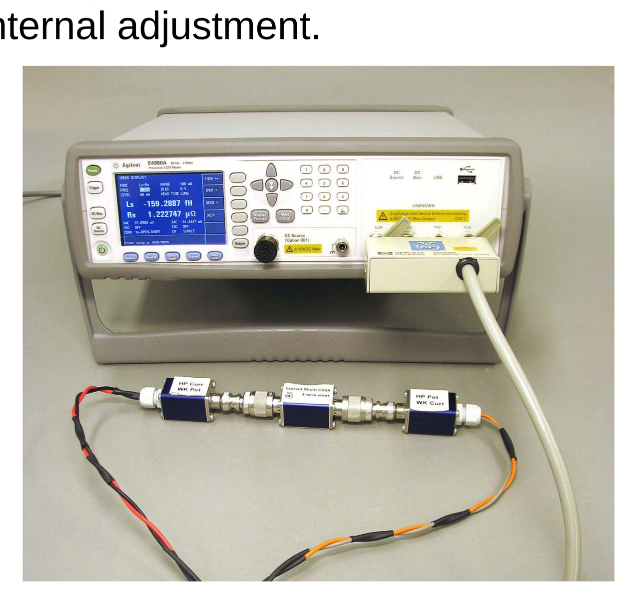



Figure 1 Repeatability of 4T short measurements.

Some 4T inductors made on PCB with a calculable value of 18 nH and a wideband resistive voltage divider are used as standards. The expanded uncertainty of the calculated value is estimated to ±5%.

## Method

The wideband resistive voltage divider, made for ac-dc transfer, with ratio 100:1 is used to scale the voltage of the inductors so the impedance meter will measure a 100 times lower inductance. The ratio of the divider is characterized to 1 MHz. The magnitude is calibrated by ac-dc transfer measurement and the phase displacement is calibrated by a new phase measuring system, see poster.

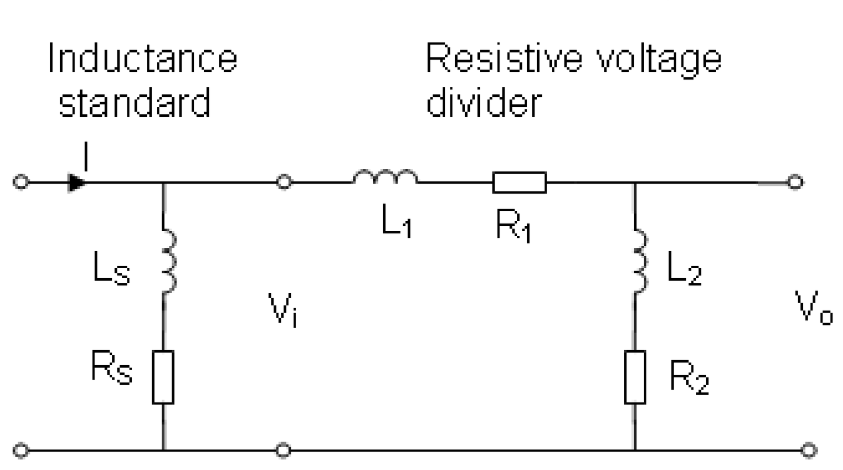



Figure 2 Circuit diagram of the inductance standard and resistive voltage divider.

The inductance standard and resistive voltage divider are modeled as shown in Figure 1. The series inductance $L_{SD}$ and resistance $R_{SD}$ of the inductance standard with divider can be determined as

$$L_{SD} = L_S D\left(1 - \frac{2R_S}{R_1 + R_2} + \frac{R_S \tau}{L_S}\right) = L_S D c \qquad (1)$$

and

$$R_{SD} = R_S D\left(1 - \frac{R_S}{R_1 + R_2} + \frac{(\omega L_S)^2}{R_S (R_1 + R_2)} - \frac{\omega^2 L_S \tau}{R_S}\right) \qquad (2)$$

with symbols according to Figure 1 and where $D$ is the ratio of the divider

$$D = \frac{R_2}{R_1 + R_2} \qquad (3)$$

and $\tau$ is a time constant determined by measuring the phase displacement ≈ωτ of the divider

$$\tau = \frac{L_2}{R_2} - \frac{L_1 + L_2}{R_1 + R_2} \qquad (4)$$

The model can be used to 1 MHz for low value inductances with low self-capacitance. The correction factor for the error due to loading of the divider, b, is so far not modeled but determined by measurement.

## Test of method

The method was evaluated by step-down measurements from 100 µH to 1 µH and to 10 nH (approx). Three standards are used: 100 µH and 1 µH standards calibrated at 1 kHz and a calculable 18 nH standard. The error of the 100 µH+100/1 divider relative the value determined by the model was measured at 0.1 kHz to 100 kHz using an impedance meter calibrated by the 1 µH standard. The error is within a few percent even when the correction for the divider is 70%, at 100 kHz. Similarly the error of the 1 µH+100/1 divider relative the value determined by the model was measured from 1 kHz to 500 kHz using an impedance meter calibrated by the calculable 18 nH standard.

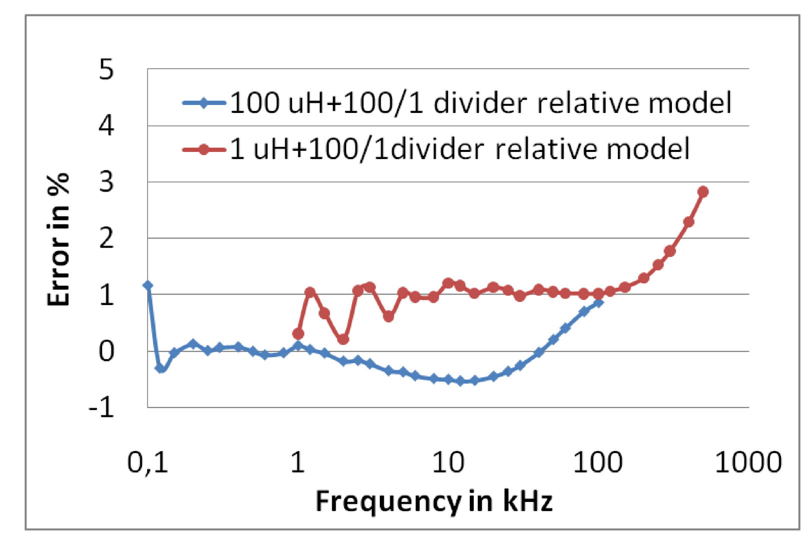



Figure 3 Test of method by measurements with "known" standards.

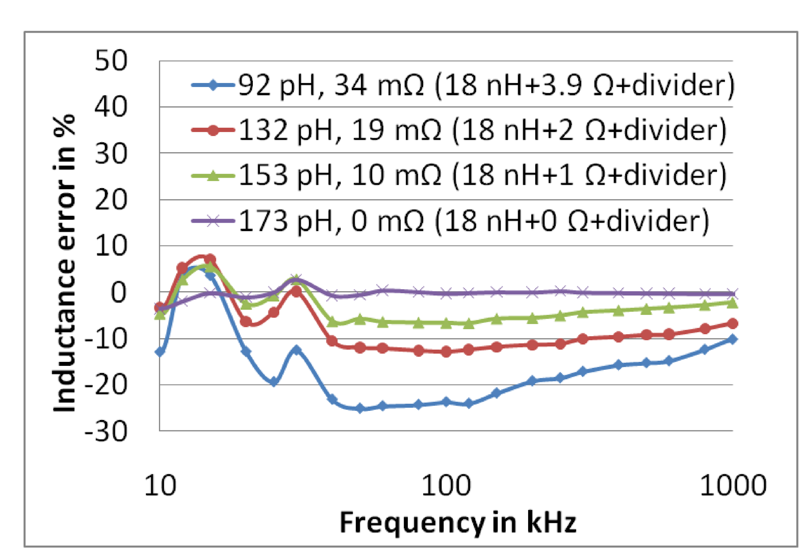
For inductance >100 nH the expressions (1) and (2) must be divided by:

$$\left(1 + \frac{(\omega(L_S + L_1 + L_2))^2}{(R_1 + R_2)^2}\right) \qquad (5)$$

## Measurements

Preliminary measurements have been done to calibrate the impedance meter at the pH-level and to determine its phase angle error. The measurements are made using inductance standards with different Q-values together with the divider. By measuring the same inductance, with and without an added resistance, the resistance-dependent inductance measurement error was calibrated at frequencies between 10 kHz and 1 MHz. The correction factor, $k$, for the phase angle error is determined as:

$$k = (L_{i0} - L_{iR} b_R / c_R)/R \qquad (6)$$



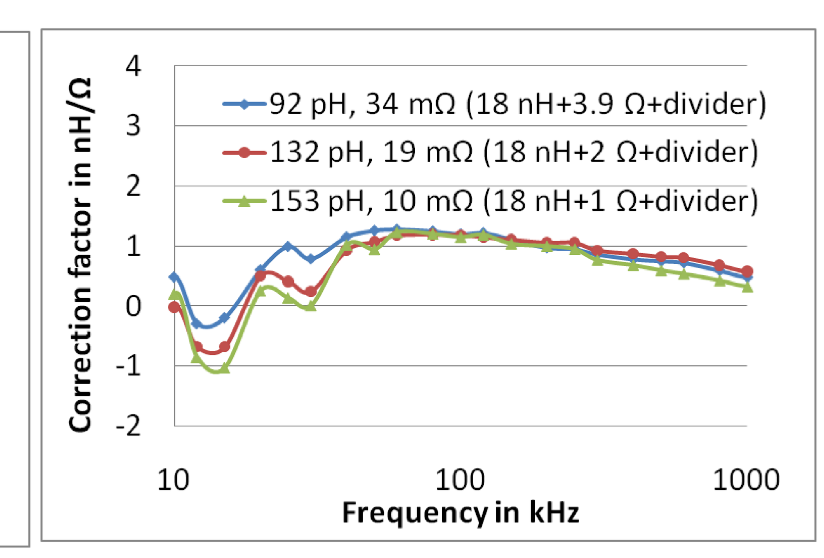




Figure 4 Inductance measuring error of the Impedance meter, 0.1 Ω range, and correction factor, *k*. Measured equivalent inductance is determined as $L_{eq}=L_i+kR$.

## Phase comparison

The table below show preliminary results of a comparison of the phase displacement of three 100 A current shunts at 100 kHz, as measured by INRIM and by SP, with and without correction for the phase error of the impedance meter. INRIM has made a realization of absolute phase of a shunt at the current level 1 A - 5 A and made a step-up using a phase comparator, see poster.

| Shunt | Resis-tance / mΩ | Phase displ. INRIM /mrad | Phase displ. SP without corr. /mrad | Phase displ. SP with corr. /mrad | Phase difference INRIM - SP (with corr) /mrad | Corres-ponding shunt inductance /pH |
|---|---|---|---|---|---|---|
| V1280A2/ V1680A5 | 5 | 0,48 | -1,11 | -0,38 | 0,86 | 7 |
| CS1D0701 | 8 | 4,10 | 2,81 | 3,55 | 0,55 | 7 |
| CS2D0702 | 8 | 4,21 | 2,90 | 3,64 | 0,57 | 7 |

The shunt V1280A2 was measured by INRIM and V1680A5 was measured by SP but both shunts are of the same model.

## Conclusion

A method to calibrate four-terminal pair inductance meters at the pH-level and at frequencies up to 1 MHz has been developed. The calibration is done using inductance standards in the nH-range and a wideband resistive voltage divider for scaling. Preliminary results looks promising.

## Acknowledgement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